

My topic this morning is “Web Mercator: Non-Conformal, Non-Mercator”

Two terms in the title require clear definition. They are “web Mercator” and “conformal”. And we’ll get right to it.

But first you should know that this entire presentation - with speaker’s notes - is posted for download at the URL given at the bottom of the screen, “www . hydrometronics . com”. We’ll do a little math this morning, but just relax. You needn’t take notes. Anything you miss can be downloaded later.

Web Mercator Defined

- Web Mercator is the mapping of WGS84 datum (i.e. ellipsoidal) latitude / longitude into Easting / Northing using spherical Mercator equations (where $R = a$)
- EPSG coordinate operation method code 1024 (Popular Visualization Pseudo Mercator, PVPM)
- EPSG CRS code 3857 WGS84/PVPM (CRS code 3785 is deprecated)
- ESRI IDs 102100 (not certain about 102113)
- Safe Software (FME) 3857, 3785, 900913, LL84

Web Mercator is the mapping of ellipsoidal latitude and longitude coordinates onto a plane using spherical Mercator equations. This projection was popularized by Google in Google Maps (not Google Earth). The reference ellipsoid is always WGS84 and the spherical radius (R) is equal to the semi-major axis of the WGS84 ellipsoid (a). That's "Web Mercator".

EPSG call Web Mercator "Popular Visualization Pseudo Mercator", operation method code 1024. Coupled with WGS84, that's coordinate reference system (CRS) code 3857. ESRI and FME have their own codes for Web Mercator given here.

Web Mercator has been called a cartographic advancement on the Internet. Today I will reveal its dark side to you.

John P. Snyder on Mercator

- Cylindrical
- Conformal
- Meridians are equally spaced straight lines
- Parallels are unequally spaced straight lines
- Loxodromes (rhumb lines) are straight lines
- Not perspective
- Poles are at infinity
- Used for navigation
- Presented by Mercator in 1569

John Snyder was the dean of American cartography in the late 20th century. Snyder presents these characteristics of the Mercator projection in both its spherical and ellipsoidal forms.

Its developable surface is a cylindrical,
It's conformal (about which we'll have much to say),
Its meridians are equally spaced straight lines,
Its parallels are unequally spaced straight lines
Loxodromes (also called rhumb lines, which are lines of constant heading) are straight lines,
It's not perspective (but mathematically derived),
Its poles are at infinity,
It's used for navigation (because rhumb lines are straight),
It was presented by Mercator in 1569.

Among these characteristics I will focus upon "conformal".

Conformal Defined

“Many of the most common and most important map projections are *conformal* or *orthomorphic* ... in that normally the shape of every *small* feature of the map is shown correctly... An important result of conformality is that relative angles at each point are correct, and the local scale in every direction around any one point is constant.”

John P. Snyder, Map Projections Used by the U. S. Geological Survey, 1983

Snyder defined conformal in this way.

Conformal projections preserve shape and angle.

Notice the portion that I've underlined, “the local scale in every direction around any one point is constant”.

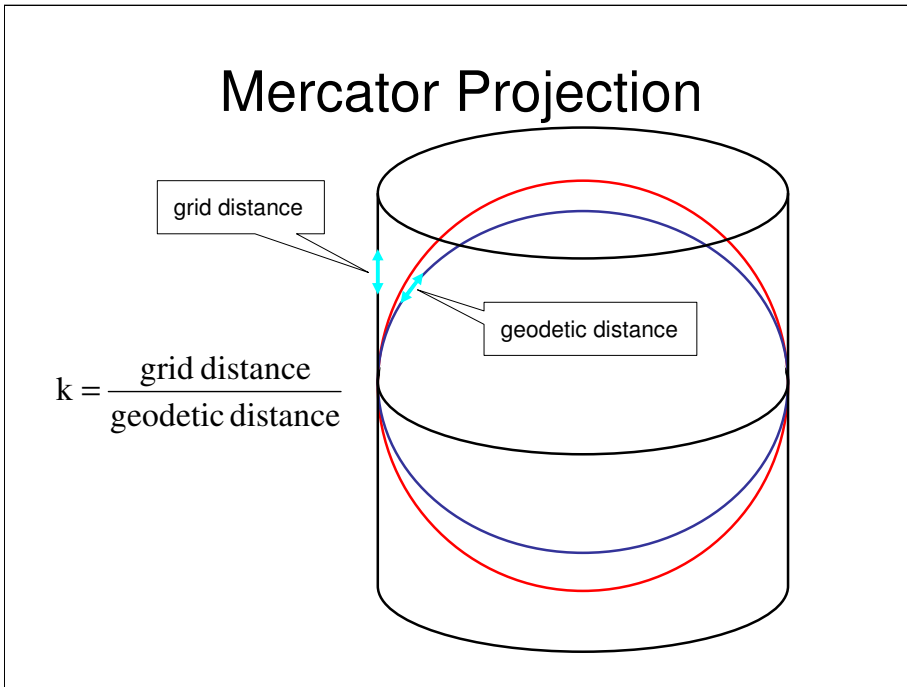
Point Scale Factor (k)

- On conformal maps the point scale factor (same in all directions) is the ratio of grid distance to geodetic distance
- $k = \text{grid distance} / \text{geodetic distance}$
- This is important for converting map distances to real-world distances quantitatively
- This applies to both the spherical and ellipsoidal Mercator
- It does not apply to the Web Mercator whose scale factor varies as a function of azimuth

This leads us to a new definition. Point scale factor (sometimes noted 'k', sometimes 'h', sometimes 'q') is an important quantity related to conformality. It is the ratio of grid distance by geodetic distance.

Point scale factor is not map scale, e.g. 1cm = 1km or 1:100,000. The grid distance here is what you get after the map scale is applied to a measured distance on the map. That grid distance differs from geodetic distance by the scale factor.

The scale factor at a point on a conformal map projection (such as the spherical Mercator or the ellipsoidal Mercator) is uniform in all directions. This is not true on a Web Mercator.



This cartoon illustrates the concept of scale factor at a point.

First, we need to remind ourselves that the Mercator is not a perspective projection. Unlike the Orthographic, for example, the Mercator cannot be constructed geometrically ... only mathematically. Nevertheless, this cartoon illustrates the scale factor concept.

The red circle represents a sphere. The blue ellipse represents an ellipsoid. The grey cylinder is the developable surface of the Mercator projection, which is rolled out into a plane.

The double-headed blue arrow on the ellipsoid represents the geodetic distance. The double-headed blue arrow on the cylinder represents the grid distance, which - on the Mercator projection - is always longer in grid meters than the geodetic distance is in true meters. The ratio of the two is the scale factor.

Spherical Mercator Equations

$E = R(\lambda - \lambda_0)$	Easting
$N = R \ln \tan(\pi / 4 + \phi / 2)$	Northing
$h = k = 1 / \cos \phi$	point scale

These E/N equations and those that follow are not important. Note, however, that **h = k**, that is N/S scale factor equals E/W scale factor. This is the desired property of conformality.

These are the equations for the spherical Mercator. Relax. For the thesis of this presentation we are only interested in the third equation, that for the point scale factor, which is represented as both 'h' and 'k'.

'h' is the scale factor in the N/S meridian direction. 'k' is the scale factor in the E/W parallel direction. $h = k$ because the scale factor at a point is the same in all directions on the spherical Mercator projection. In other words, the spherical Mercator is conformal.

Ellipsoidal Mercator Equations

$$E = a(\lambda - \lambda_0) \quad \text{Easting}$$
$$N = a \ln \left[\tan(\pi/4 + \phi/2) \left(\frac{1 - e \sin \phi}{1 + e \sin \phi} \right)^{e/2} \right] \quad \text{Northing}$$
$$h = k = (1 - e^2 \sin^2 \phi)^{1/2} / \cos \phi \quad \text{point scale}$$

$$\mathbf{h = k}$$

WGS84 semi-major axis (a) is 6378137m

WGS84 eccentricity (e) is 0.081819191

These are the equations for the ellipsoidal Mercator. They're a little more complicated, especially in Northing. The parameters 'a' (semi-major axis) and 'e' (eccentricity) are given for the WGS84 ellipsoid. But again for the thesis of this presentation we are only interested in the third equation, that for the point scale factor, which is represented as both 'h' and 'k'.

Again, 'h' is the scale factor in the N/S meridian direction. 'k' is the scale factor in the E/W parallel direction. $h = k$ because the scale factor at a point is the same in all directions on the ellipsoidal Mercator projection. In other words, the ellipsoidal Mercator is conformal.

Web Mercator Equations

$E = a(\lambda - \lambda_0)$	Easting
$N = a \ln \tan(\pi/4 + \phi/2)$	Northing
$q(\alpha) = [a / R'(\alpha)] / \cos \phi$	scale(α)
Where,	
$R'(\alpha) = \rho\nu / (\nu \cos^2 \alpha + \rho \sin^2 \alpha)$	radius(α)
$\rho = a(1 - e^2) / (1 - e^2 \sin^2 \phi)^{3/2}$	radius meridian
$\nu = a / (1 - e^2 \sin^2 \phi)^{1/2}$	radius prime vertical
$h = q(0, 180) = (a / \rho) / \cos \phi$	scale N/S
$k = q(90, 270) = (a / \nu) / \cos \phi$	scale E/W

h <> k, therefore non-conformal

These are the equations for the Web Mercator. With the substitution of 'a' for 'R', the equations for Easting and Northing are the same as for the spherical Mercator. But what's happened to the single equation for the scale factor at a point? It has escalated into several, more-complex equations!!!

The scale factor at a point is now called 'q' because it is different for every direction. 'q' is a function of R', the radius of curvature of the ellipsoid in the direction alpha. R' is in turn a function of rho and nu, the radii of curvature in the meridian and the prime vertical respectively, and the direction alpha.

For the Web Mercator 'h' and 'k' as given here are not equal. Thus, the Web Mercator is not a conformal projection.

Computational Efficiency

Relative times for 1M conversions based on
 spherical forward without scale (black)
 spherical reverse without scale (blue)

FORWARD		REVERSE		
Spherical E/N only	1	Spherical L/L only	2.7	1
Spherical E/N + scale	2.3	Spherical L/L + scale	4	1.5
Ellipsoidal E/N only	4.9	Ellipsoidal L/L only	14.3	5.3
Ellipsoidal E/N + scale	9	Ellipsoidal L/L + scale	15.8	5.9
Web E/N only	1	Web L/L only	2.9	1.1
Web E/N + scale	12.4	Web L/L + scale	13.9	5.2

Web Mercator is not faster if scale is computed

One often-cited advantage of the Web Mercator is its computational efficiency.

These tables of relative times for 1 million computations each show that the Web Mercator is as fast as the spherical Mercator for Easting/Northing (forward) and Latitude/Longitude (reverse). Both the spherical Mercator and Web Mercator are about 5 times faster than the ellipsoidal Mercator for both the forward and reverse conversions.

But these tables also show that if scale factor at a point is added to the conversion (as a surveyor might require), then the Web Mercator is much slower than the spherical Mercator and even a little slower than the ellipsoidal Mercator. And that slowness delivers the scale factor in only one direction. If more directions are required, it's slower and slower!

So what?!?

The Web Mercator is just for
visualization on the web!

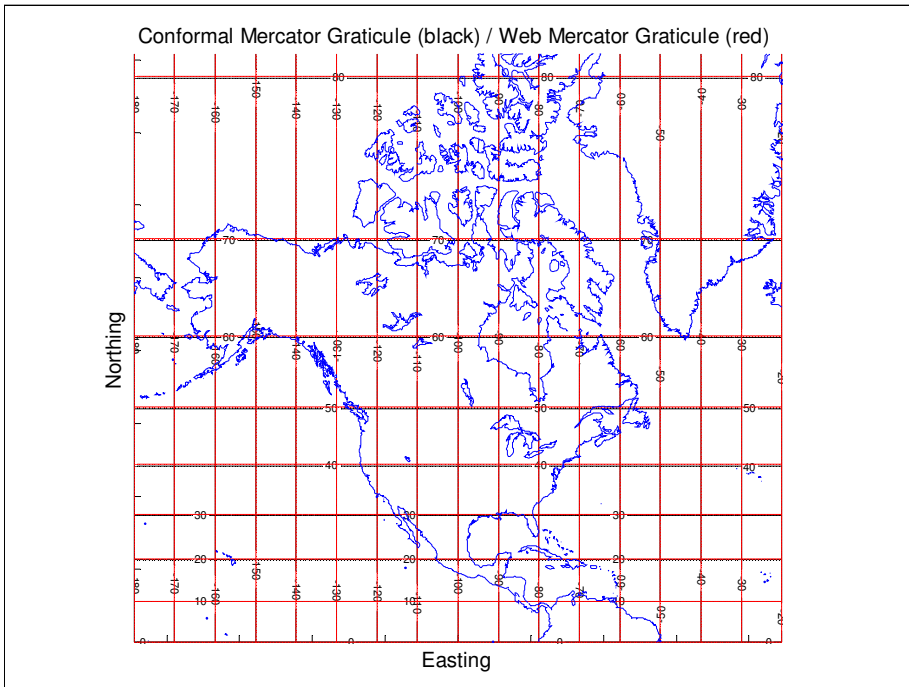
Or is it?

So what?!? The Web Mercator is certainly fit for printing out directions to a new restaurant across town or for visualization on your computer screen or for other purposes on the web.

But the Web Mercator is a projection that has jumped from one species (the web) to another species (GIS) where it's leading another life. Witness the EPSG, ESRI and FME codes for the Web Mercator.

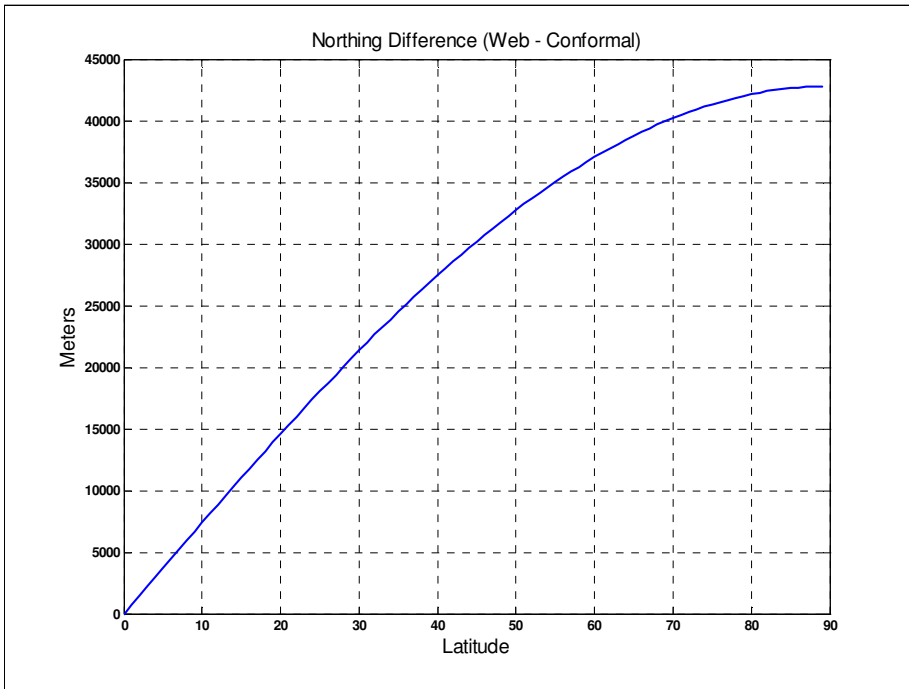
Surveyors and GIS professionals need to know that the Web Mercator is not conformal. If distance computations on the Web Mercator are done simply (as they can be done on a conformal projection), they will be wrong. If done correctly, they will be laborious.

Some differences between the Web Mercator and the ellipsoidal Mercator are quantified on the next few slides.



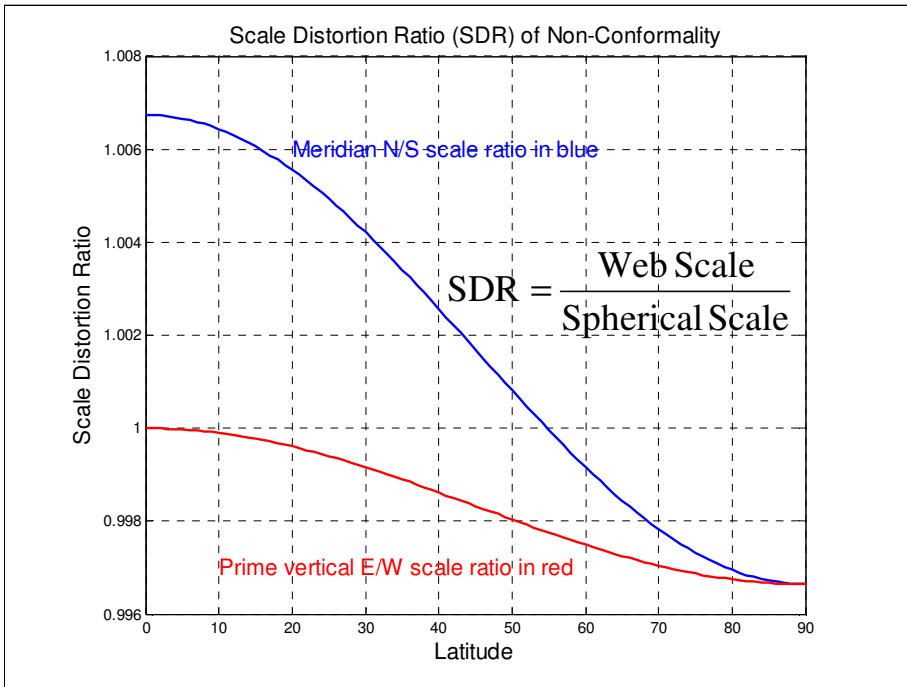
For an area the size of the NW quadrisphere (North America) the differences appear slight.

It turns out that the Eastings are identical. The differences are in the Northings.



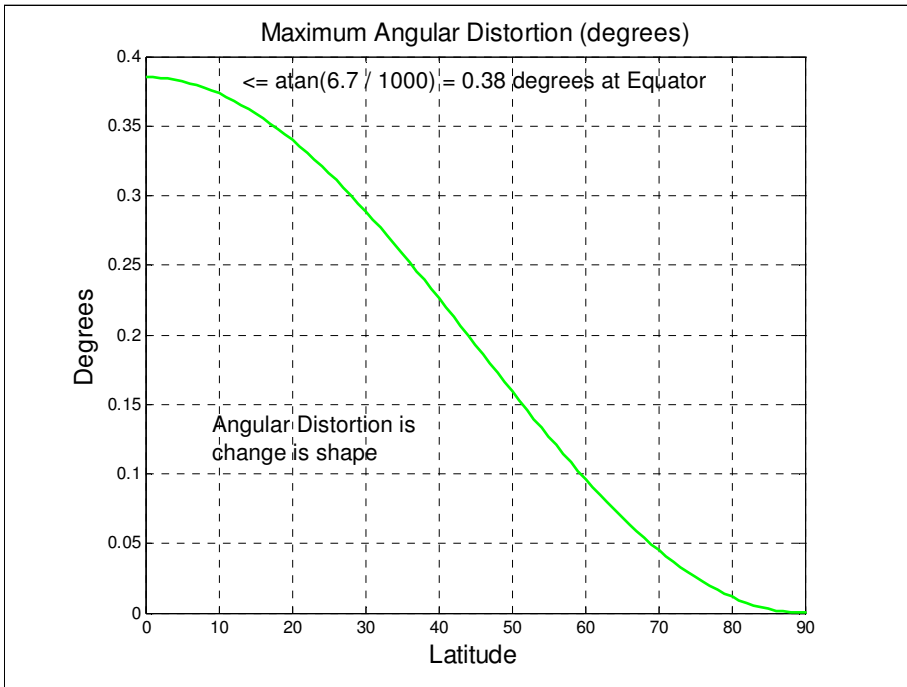
There is no Northing difference at the Equator, but by 70 degrees North, the difference is 40km.

This NS stretching in the Web Mercator is the reason for its non-conformality.



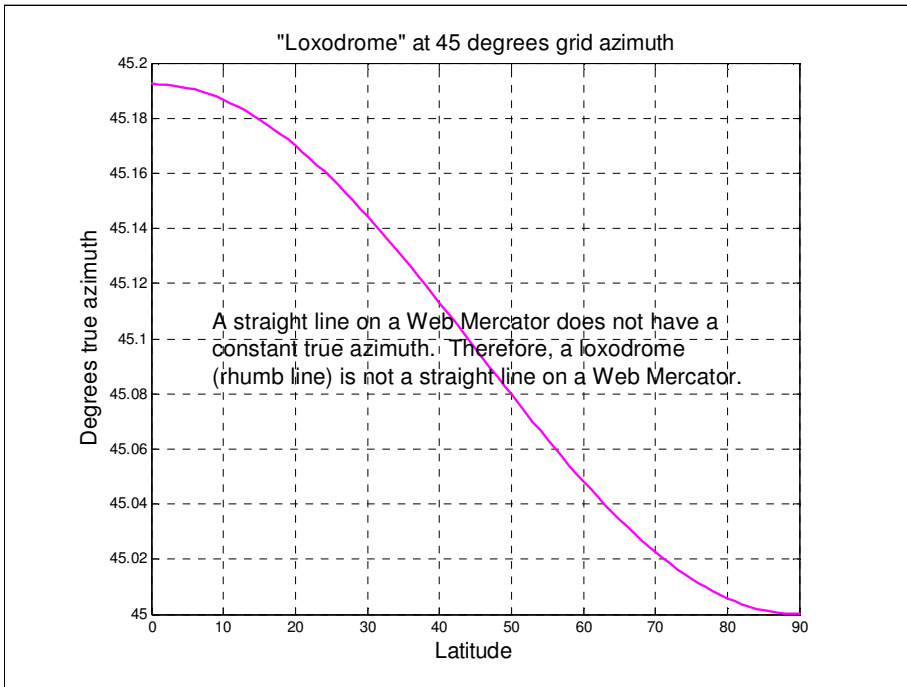
If a naïve user expects that the Web Mercator – which uses the spherical Mercator equations – has the same scale distortions as the spherical Mercator, then the Scale Distortion Ratio (SDR, the author’s invention) is the proportional error that will be made.

At the Equator it’s +6.7m per km in the N/S direction (blue) but 0m in the E/W direction (red). It varies with latitude as shown down to -3m per km in both directions approaching the poles.



Another feature of a conformal projection is the preservation of shape in a local area (called orthomorphism). Non-conformal projections such as the equal-area Albers, for example, distort shape, and so does the Web Mercator. Maximum angular distortion is a measure of this shape shifting.

At the Equator the maximum angular distortion is about four tenths of a degree, and it decreases to zero as we approach the pole.



Mercator projections are useful for navigation because rhumb lines are straight. These are lines of constant true heading that navigators used to sail before GPS.

Straight lines on a Web Mercator are not rhumb lines and this plot quantifies the difference as a function of latitude.

Web Mercator: True / False

- Cylindrical (yes)
- Conformal (no)
- Meridians are equally spaced straight lines (yes)
- Parallels are unequally spaced straight lines (yes, but in a different way than a conformal Mercator)
- Loxodromes (rhumb lines) are straight lines (no)
- Not perspective (yes, not perspective; mathematical)
- Poles are at infinity (yes)
- Used for navigation (doubtful)
- Presented by Mercator in 1569 (no, presented by Google in the 21st century)

So, to summarize we return to Snyder's characteristics of the Mercator and see which is true for the Web Mercator.

Yes, the Web Mercator is cylindrical.

No, it is not conformal.

Yes, its meridians are equally spaced straight lines.

Yes, its parallels are unequally spaced straight lines, but in a different way than a conformal Mercator.

No, its loxodromes (rhumb lines) are not straight lines.

Yes, it is not perspective.

Yes, its poles are at infinity.

It's doubtful that it's used for navigation.

And, no, it was not presented by Mercator in 1569, but by Google recently.

Concluding Syllogism

Mercator projection is conformal.

Web Mercator is non-conformal.

Therefore, Web Mercator is not a Mercator projection.

Corollary:

If you need to use the Web Mercator quantitatively (i.e. other than for visualization), you'll pay a price for its deceptive simplicity

I conclude with this syllogism and its corollary.

The Mercator projection is conformal.

The Web Mercator is non-conformal.

Therefore, the Web Mercator is not a Mercator projection.

The corollary is that:

If you need to use the Web Mercator quantitatively (i.e. other than for visualization), you'll pay a price for its deceptive simplicity

References are next.

References

- EPSG Guidance Note 7, Part 2, Section 1.3.3.2
 - www.epsg.org
- Discussion on the Proj4 listserver with great contributions by Mikael Rittri and daan Strebe (who have no responsibility for anything I say)
 - <http://lists.maptools.org/pipermail/proj/>
- This paper with speaker's note is available for download at the author's website
 - www.hydrometronics.com

EPSG Guidance Note 7 Part 2 Section 1.3.3.2 gives all the mathematics for the Popular Visualization Pseudo Mercator.

The Proj4 list server has discussed this topic in great length. Those cited helped me form my opinions in this matter.

This paper with speaker's notes is available at www.hydrometronics.com

Appendix

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Reverse Spherical Mercator

$$\lambda = E / R + \lambda_0 \quad \text{longitude}$$

$$\phi = \pi / 2 - 2 \arctan(\exp(-N / R)) \quad \text{latitude}$$

Reverse Ellipsoidal Mercator

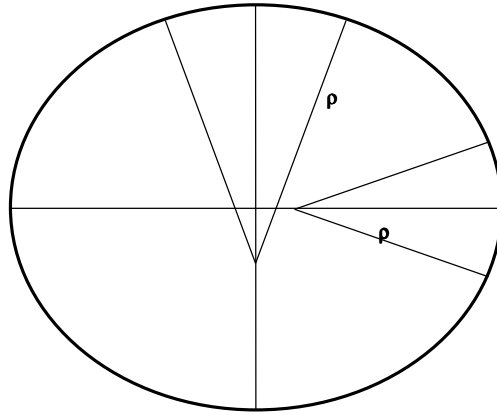
$$\lambda = E / a + \lambda_0$$

$$\phi = \pi / 2 - 2 \arctan \left\{ \exp(-N / a) \left[\frac{(1 - e \sin \phi)}{(1 + e \sin \phi)} \right]^{e/2} \right\}$$

Latitude is solved iteratively. Seed with spherical.

For completeness the reverse equations for both the spherical and ellipsoidal Mercators are given. Web Mercator uses the spherical equations.

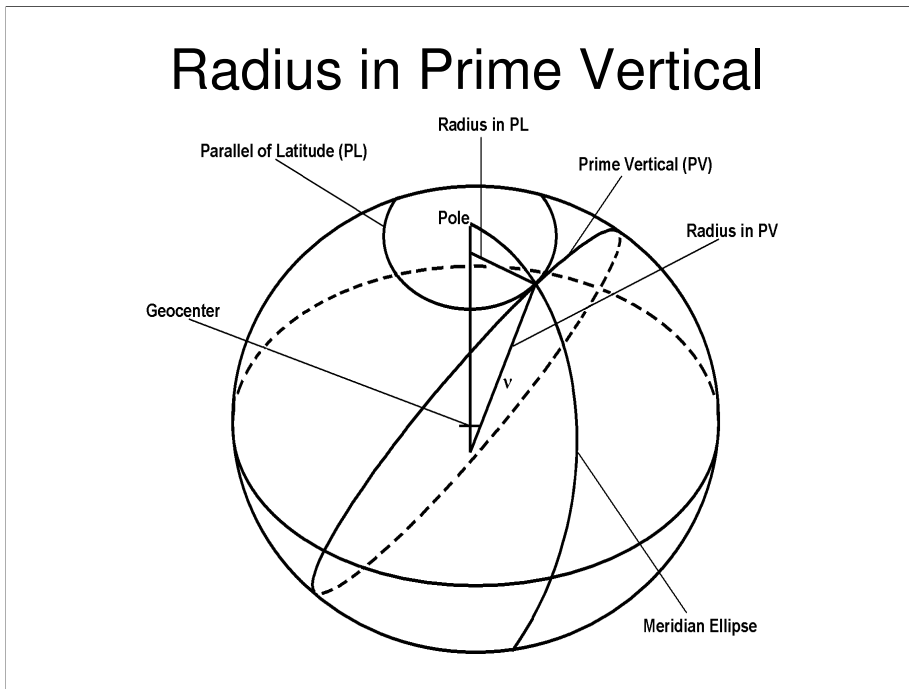
Radii in the Meridian



So, what are these radii of the curved Earth required to compute the scale factor in the Web Mercator?

This slide exhibits the varying radius of curvature in the meridian arc, which is the N/S ellipse of constant longitude. The radius of curvature in the meridian is called “rho”. Its length varies as a function of latitude.

Radius in Prime Vertical



The E/W direction is somewhat more complicated.

The prime vertical is the great circle in the E/W direction. It crosses the Equator. The radius of curvature in the prime vertical is called “nu” (and it’s shown here).

The parallel is the small circle of constant latitude in the E/W direction. The radius of curvature in the parallel of latitude is related to the radius of curvature in the prime vertical by the cosine of latitude.

Mini Bio of Noel Zinn

- Noel Zinn began Hydrometronics LLC this year as a technical software consultancy
- Geodesist for ExxonMobil in the Naughts
- Navigation Scientist for Western Geophysical in the Nineties
- Surveyor for NCS International in the Eighties
- Navigator for Delta Exploration (Singapore) in the Seventies
- Peace Corps Volunteer in India in the Sixties
- Studied at the University of California (Berkeley) and the University of Houston (still there!)

Noel Zinn's professional bio.



Hydrometronics LLC

Hydrometronics provides consultancy and technical software development for seismic navigation, ocean-bottom positioning, subsea survey, geodesy, cartography, 3D visualization (ECEF) and wellbore-trajectory computation.

www.hydrometronics.com

I leave you with this advertisement.